3·0 cm.) using a 1·0 mm. collimator. Here the pattern is rather irregular, and does not show very obvious association of the central beam pattern with strong Laue spots. This diamond is one which shows the anomalous X-ray reflexions (Lonsdale, 1942) very strongly, but a central-beam pattern is also observed for diamonds which do not do so.

Fig. 4, however, which was obtained with a brown diamond from Premier Mine, South Africa (British Museum (Natural History); 1910, no. 227) shows a marked correlation between the Laue pattern and the central-beam pattern. Presumably in this position the strong Laue spots really do correspond to the most strongly reflecting planes.

It should perhaps be mentioned that this pattern is not due in any way to the beam-trap itself. For precisely the same experimental arrangement, the pattern is quite different for different diamonds, and alters for the same diamond when the orientation is altered. It remains the same, however, for the same diamond in the same setting with a different beam-trap.

For the diamond plate used in Fig. 3, a slight improvement in the clarity of the pattern resulted when the radiation was filtered with nickel foil, which suggests that the pattern is due to the characteristic Cu $K\alpha$ radiation, whereas on a Laue photograph multiple reflexion would result for any wavelength actually producing a Laue spot (though of course filtering also reduces the continuous radiation to some extent). This question might be cleared up by taking comparative photographs with the crystal in the Bragg position and a few degrees away from it, or by using the same setting and different radiations. Further experiments with monochromatic radiation are also required, but it is not possible for the author to continue this work, and these preliminary results are therefore placed on record.

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A method of deriving crystal geometry from random rotation photographs.

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In the course of investigations of the nature of crystalline inclusions in diamond (where it is often difficult to obtain rotation photographs of the inclusion orientated with a prominent zone axis as rotation axis) an attempt was made to interpret rotation photographs of a crystal in a random orientation by a method analogous to the general solution of powder photographs from crystals of any symmetry applied by Ito (1949).

The values of $1/d^2_{hkl}$ for all the reflecting planes in the crystal can be obtained from random rotation photographs just as they can be found from a powder photograph. The reflexions can in general be indexed in terms of an arbitrary unit cell defined by the three planes of largest spacing. If these are unsuitable for some reason, such as that they are all orders of the same reflexion, or that all three planes lie in the same zone, then combinations involving the next one or two planes will be necessary. The unit cell obtained in this way will not, however, necessarily exhibit the symmetry of the Bravais lattice of the crystal, and the problem is now to discover what that symmetry really is. This can be done graphically from the arbitrary cell, but can also be done—in terms of an elegant reduction process devised by Delaunay (1933)—quite mechanically by means of a tetrahedral diagram (a paper on this subject is in the course of preparation).

If a single crystal of the substance, and not merely a powder, is available, a very powerful modification of this method can be employed by making use of multiplicities. When a rotation photograph is taken of a single crystal in a random orientation, each spot will in general be produced by reflexion from one side of one set of planes, and thus the number of spots on any Debye ring of constant θ will be equal to (or less than) the multiplicity of the form of planes $\{hkl\}$ producing them. Theoretically it should be possible to obtain the multiplicities of various reflexions by comparing the intensities on a rotation photograph and on a powder photograph of the same crystal.

The system, and indeed the Laue group, to which a crystal belongs can be determined almost by inspection. On a rotation photograph of the usual type, recorded on a cylindrical film whose axis coincides with the axis of rotation, any plane (hkl) produces two spots above the equator, while $(\bar{h}\bar{k}\bar{l})$ produces two spots below the equator. Hence the multiplicities on one half of the film (the division being taken in practice parallel to the rotation axis) correspond to those conventionally listed in tables. Accidental coincidences of spacing can usually be detected by differences of intensity among the spots on a single Debye ring, due allowance being made for the increase of intensity on moving away from the equator (Cox & Shaw, 1930).

If there is no multiplicity greater than two, then the crystal is triclinic. For the monoclinic system there will only be multiplicities of two and four, whereas in the orthorhombic system, eight will appear, and so on. If the planes are then arranged in groups of the same multiplicity, relationships can be found much more easily than when all the reflexions from the crystal are considered together.

This method has an advantage over the general powder method proposed by Ito in that the preliminary grouping of planes in this way makes it possible to study more complicated crystals without having especially accurate $\sin^2\theta$ values. It is further of considerable value in cases where space-group extinctions cause the systematic absence of low-order reflexions from the axial planes. When in monoclinic or orthorhombic crystals the plane of largest spacing has a multiplicity higher than two, this is an immediate indication of the existence of such extinctions.

In certain crystal classes, planes of the same form $\{hkl\}$ do not all have the same numerical value of $|F(hkl)|$, and for example $|F(hkl)| \not\equiv |F(\bar{h}kl)|$, say; hence the intensities of spots on the same ring may differ just as though two different forms $\{h_1k_1l_1\}$ and $\{h_2k_2l_2\}$ accidentally had the same spacing. In practice, however, a general consideration of the multiplicities present would indicate the system to which the crystal belonged, and a study of a few planes of low multiplicity and large spacing should give a unit cell from which the remaining rings can be indexed.

A limited application of the use of multiplicities was recorded by Lipson & Steeple (1951), who found that indications of the value of the multiplicity factor on powder photographs on which the lines were 'spotty' is a useful aid to indexing.

When using the method, we have taken two photographs of each crystal, one in a random orientation and a second exposure for which one arc was altered 55° and the other arc 10° with respect to the first photograph, so that a reasonable amount of the limiting sphere was covered in all, and important planes which might be missed on one photograph would probably appear on the other. Quarter-plate films were used, one about 2·5 cm. from the crystal in order to obtain a large number of reflexions, and one approximately 7·5 cm. from the crystal for more accurate measurement of small θ values. Rotation photographs on flat films are more convenient than those on cylindrical films, since circles can then be drawn through equivalent points.

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Transformation properties of the physical constants of crystals.

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Material constants connecting physical quantities in crystals, for instance, the elastic constants, are usually given with respect to an orthogonal co-ordinate system the orientation of which is determined by crystal symmetry. Various methods have been developed to express the material constants proper to an arbitrary co-ordinate system in terms of those above, but the procedures are usually involved (see for example Cady, 1946).

We would like to call attention to a compact method making use of the transformation properties of single co-ordinate products. The method is applicable to transformations between orthogonal co-ordinate systems. Here we need not distinguish between co- and contra-variant quantities.

The material constants connecting two tensors of rank m and n transform essentially like the components of a tensor of rank $(m+n)$. These transformation properties are well known. If the co-ordinates of the same point, referred to two co-ordinate systems, are connected by

$$x'_i = \Sigma R_{ij} x_j, \qquad (1)$$

where $\Sigma R_{ij} R_{ik} = \delta_{jk}$, an ordinary tensor transforms like

$$T'_{ij}\ldots = \Sigma R_{is} R_{jt} \ldots T_{st} \ldots. \qquad (2)$$

Equation (2) is not affected by symmetry in the tensor indices if the summation is carried out over all possible values of the summation indices.

Each of the tensor indices can have three possible values. Hence we can say that a tensor component transforming according to (2) transforms like the co-ordinate product

$$(x_i)_1 (x_j)_2 \ldots, \qquad (3)$$

where the numerical subscripts refer to the position of the various indices of the tensor. We can find $T'_{ij}\ldots$ by using (1) on each co-ordinate appearing in (3), and by then identifying each term in the resultant sum of co-ordinate products with the proper tensor component in the original co-ordinate system.

In principle, it is no more difficult to carry out the transformation of the material constants according to (2) than to identify them with co-ordinate products (3) and then carry out the transformation on these co-ordinate products. But the use of co-ordinate products has some practical advantages.

Tensors representing physical quantities are often symmetric in some indices. This symmetry can be incorporated in the co-ordinate products by using the same numerical subscript on the co-ordinates representing interchangeable tensor indices. Thus, the components of the stress-tensor transform like the products

$$x^2,\quad y^2,\quad z^2,\quad yz,\quad zx,\quad xy; \qquad (4)$$

that is, no identifying subscripts are needed.

Since material constants transform like the product of the tensor components they connect, they transform like the products of the co-ordinate products representing the tensor components in question. Any symmetry in the tensor indices can thus be carried over directly into the appropriate parts of the total co-ordinate product. This reduces the number of subscripts of (3) and hence the number of products upon expansion after a transformation. Additional symmetry in the material constants, such as index-pair symmetry, is best included after the transformation has been carried out.